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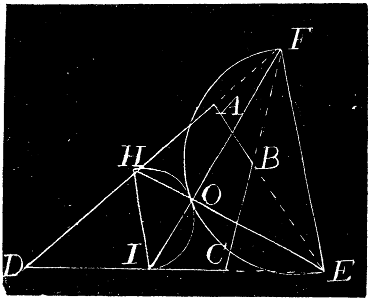

Draw EH and FI to bisect the angles E and F, and let O be the point of their intersection. Draw, also, the diagonals AC, BD. Then, evidently, we have

$$\angle ABC = BFO + FOE + OEB$$
$$= \tfrac{1}{2}\angle F + FOE + \tfrac{1}{2}\angle E,$$
$$\angle ADC = OIC - \tfrac{1}{2}\angle F$$
$$= FOE - \tfrac{1}{2}E - \tfrac{1}{2}F$$

$\therefore$ $\angle ABC + ADC = 2FOE =$ two right angles when FOE is a right angle, and the points A, B, C, D will be on the circumference of a circle. Hence this criterion:—

Prolong the opposite sides AB, DC and DA, CB to meet in E and F. Draw the bisectors EH; FI, and join IH, EF, upon which describe semicircles. Then, if these semicircles touch each other in the point of intersection (O) of the bisectors, FO will be perpendicular to EO, angles $A+B$ will $=$ angles $B+D$, and the quadrangle can be circumscribed by a circle.

ON THE ROOTS OF EQUATIONS.

BY PROFESSOR WORPITZKY, BERLIN, PRUSSIA.*

SUPPOSE a portion of an area z to be ascertained which contains a single root of the equation $f(z) = 0$; the root itself $z = c$ being unknown.

Morover, the function, $f(z)$, is supposed to be monogen in all parts of the area; finally determined; and with the exception of $z = c$ to be nowhere equal to zero.

Then $f(z) = (z - c)\varphi(z)$ may be substituted, in which equation the function, $\varphi(z)$, possesses exactly the same attributes, except that $\varphi(z) = C$; that is a finite value different from c: and after the well known formula of *Lauch* concerning marginal integrals we obtain

$$\frac{2\pi i}{C} = \int^{c} \frac{dz}{f(z)}.$$

Hence C may be ascertained without knowing the value of c.

On the other hand;

$$f'(z) = \varphi(z) + (z - c)\varphi'(z);$$
$$f'(c) = \varphi(c) = C.$$

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*As this paper from Prof. Worpitzky, in German, was accompanied by a letter to Alex. Evans, Esq., of Elkton, Md., referring to his translation at p. 66, Vol. IV, ANALYST, the original was sent by us to Mr. Evans, to whom we are indebted for this translation.—Ed.

Hence every single root c, *of the equation* f(z) = 0, *is also a root of the equation* f′(z)—C = 0, *if the true value, viz.,*

$$C = \frac{2\pi i}{\int^{c} \frac{dz}{f(z)}},$$

be substituted for C.

The fencing in, (*einhegung*), or determination of each single root *c*, and the numerical calculation of the marginal integrals around the areas in which they lie, need not be explained here, as their practical admissibility (*ausfurbarkeit*) is theoretically unquestioned.

Nor need the train of consequences, which might be drawn from the foregoing, be enumerated here, the following only need be taken into consideration. (*platzfinden.*)

The processes (*mittel*) are known by which to reduce one *algebraic* equation to another, to wit, to $f(z) = 0$, which contains all the roots of the former as single roots; we also know how to determine (*einzuhegen*) each one of the latter.

Whenever this is done our formula in question presents the root $z = c$ as a root of the equation $f'(z) - C = 0$, which is lower than the former by one degree.

This equation may then be treated in like manner as the former, *and then we arrive, by a finite number of entirely definite and practical operations, at an equation of the first degree, viz.*; z = c, *in which the value of* c *is composed of the marginal integrals in a form similar to a continued fraction.*

Moreover we obtain z—c *as a common divisor of* f(z) *and of* f′(z)—C.

RECTIFICATION OF THE HYPERBOLA.

BY ARTEMAS MARTIN, M. A., ERIE, PA.

THE equation to the hyperbola, in rectangular coordinates referred to its center, is

$$b^2x^2 - a^2y^2 = a^2b^2. \tag{1}$$

Denoting by H_x the arc whose abscissa is x,

$$dH_x = \left(1 + \frac{dy^2}{dx^2}\right)^{\frac{1}{2}} dx = \left(\frac{[1 + (b^2 \div a^2)]x^2 - a^2}{x^2 - a^2}\right)^{\frac{1}{2}} dx,$$

$$= \left(\frac{e^2x^2 - a^2}{x^2 - a^2}\right)^{\frac{1}{2}} dx = a\left(\frac{e^2v^2 - 1}{v^2 - 1}\right)^{\frac{1}{2}} dv,$$

where e, the eccentricity, $= \frac{(a^2 + b^2)^{\frac{1}{2}}}{a}$, and $av = x$.